UNIT - 1 MATRICES

ESSAY QUESTIONS

1) Define an orthogonal Matrix and solve

$$A = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix}$$

2) Show that every square Matrix can be expressed as a sum of Hermition and skew Hermition matrices in one and only way.

3) Show that $\begin{bmatrix} a+ic & -b+id \\ b+id & a-ic \end{bmatrix}$ is unitary $a^2+b^2+c^2+d^2=1$

4) Find the value of "k" if the rank of matrix A is 2

where $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & K & 0 \end{bmatrix}$

5) a) Find the rank of the Matrix $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ by reducing to echelon form

b) Find the rank of $\begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ -2 & -3 & -1 & 4 & 3 \\ -1 & 6 & 7 & 2 & 9 \\ -3 & 3 & 6 & 6 & 12 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ -2 & -3 & 1 & 2 \\ -3 & -4 & 5 & 8 \\ 1 & 3 & 10 & 14 \end{bmatrix}$ Find the rank

6. Find the rank of $\begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ 2 & -3 & -1 & 4 & 3 \\ -1 & 6 & 7 & 2 & 9 \\ -3 & 3 & 6 & 6 & 12 \end{bmatrix}$

7. Find the rank of $\begin{bmatrix} 1 & 2 & 3 & 4 \\ -2 & -3 & 1 & 2 \\ -3 & -4 & 5 & 8 \\ 1 & 3 & 10 & 14 \end{bmatrix}$

8) What is Normal form of a Matrix and find the rank of given Matrix $A = \begin{bmatrix} 2 & 1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$

9) By reducing the matrix $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & +1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ into Normal form and find it's rank

10) $A = \begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$ reduce to Normal form and find its rank.

16) Find the value of λ for which the system of equation $3x-y+4z=3$, $x+2y-3z=-2$, $6x+5y+\lambda z=-3$ will have infinite number of solutions and solve them with that λ value.

17) Prove that the following set of equations are consistent and solve them. $3x+3y+2z=1$, $x+2y=4$, $10y+3z=-2$, $2x-3y-z=5$

18) Solve the system of equations

$x+2y+3z=1$, $2x+3y+8z=2$, $x+y+z=3$

19) Find the values of P and Q so that the equation $2x+3y+5z=9$, $7x+3y+2z=8$, $2x+3y+Pz=q$ have
i, No solution ii, unique solution iii, An infinite no. of solution.

20) Find whether the following system of equations are consistent.
If so, solve them.

$x+2y-z=3$, $3x-y+2z=-1$, $2x-2y+3z=2$, $x-y+z=-1$

21) Determine whether the following equations will have a solution. If so, solve them.

$x_1+2x_2+x_3=2$ $\quad$ $3x_1+x_2-2x_3=1$

$4x_1-3x_2-x_3=3$ $\quad$ $2x_1+4x_2+2x_3=4$

11) Find the inverse of the Matrix A using elementary operation (Gauss - Jordan Method)

i) $A = \begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$

ii) $B = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$

12) Solve the system of linear equations by Matrix Method $x+y+z=6$, $2x+3y-2z=2$, $5x+y+2z=13$?

13) If $A = \begin{bmatrix} 3 & 7-4i & -2+5i \\ 7+4i & -2 & 3+i \\ -2-5i & 3-i & 4 \end{bmatrix}$ then show that A is Hermitition and iA is skew Hermitition Matrix.

14) Discuss for what values of λ, μ is simultaneous equations $x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$ have

i) No solution

ii) an unique solution

iii) an infinite number of solution

15) Find whether the following system of equations are consistent. If so solve them

[$x+2y+2z=2$, $3x-2y-z=5$, $2x-5y+3z=-4$, $x+4y+6z=0$]

22) Show that the equation $3x+4y+5z=a$, $4x+5y+6z=b$ and $5x+6y+7z=0$ do not have a solution unless at $a+c=2b$.

23) Solve completely the system of equations

$x+y-2z+3w=0$ $\quad x-2y+z-w=0$

$4x+y-5z+8w=0$ $\quad 5x-7y+2z-w=0$.

24) Test for consistency and if consistent solve the system,

$5x+3y+7t=4$, $3x+26y+2t=9$, $7x+2y+10t=5$

25) Solve the system of equations

$x+y+w=0$, $y+z=0$, $x+y+z+w=0$, $x+y+2z=0$.

26) Examine whether the vectors are linearly dependent or not $(3,1,1)$, $(2,0,-1)$, $(4,2,1)$

27) Determine the values of λ for which the following set of equation may possess non-trivial solution

$$3x_1+x_2-\lambda x_3=0$$
$$4x_1-2x_2-3x_3=0$$
$$2\lambda x_1+4x_2+\lambda x_3=0$$

For each permissible value of λ, determine the general solution.

28) show that the only real number λ for which the system $x+2y+3z=\lambda x$; $3x+y+2z=\lambda y$; $2x+3y+z=\lambda z$ has non-zero solution is 6 and solve them, when $\lambda=6$

29) use gauss-elimination method to solve $x+2y-3z=9$, $2x-y+z=0$, $4x-y+z=4$

30) Solve the following system of equations by using Gauss-seidal Method. and correct to three decimal places.

i) $8x-3y+2z=20$

$4x+11y-z=33$

$6x+3y+12z=35$

ii) $x_1+10x_2+x_3=6$

$10x_1+x_2+x_3=6$

$x_1+x_2+10x_3=6$.

UNI-I

SHORT ANSWER TYPE:-

1) Define ~~skew~~ symmetric and skew symmetric matrix and give an example

2) Define conjugate of a Matrix.

3) Define Hermitian and skew-Hermitian Matrix

4) Define an unitary Matrix and Prove that $\frac{1}{2}\begin{bmatrix} 1+i & -1+i \\ 1+i & 1-i \end{bmatrix}$ is a unitary Matrix

5) Define the rank of Matrix

6) Define Echelon form of a Matrix.

7) Find the value of 'α' show that the vectors (1,1,0) (1,α,0) and (1,1,1) are linearly dependent.

8) Determine whether the vectors (1,2,3), (2,3,4), (3,4,5) are linearly dependent or not.

9) Express the following system in matrix form and solve by guass-elimination Method.

$2x_1+x_2+2x_3+x_4=6$; $6x_1-6x_2+6x_3+12x_4=36$;

$4x_1+3x_2+3x_3-3x_4=-1$; $2x_1+2x_2-x_3+x_4=10$

~~10)~~ ~~If~~ A

10) Prove that the transpose of a unitary Matrix is unitary.

11) Prove that the Product of two unitary Matrices is unitary.

Unit-II

Eigen values and eigen vectors.

SHORT ANSWERS

1 Define characterstic equation?

2\. Find the charecteristics roots of matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

3\. Find the sum and product of eigen values of matrix $\begin{bmatrix} 2 & 1 & -1 \\ 3 & 4 & 2 \\ 1 & 0 & 2 \end{bmatrix}$

4\. Define Hermitian and skew Hermitian

5\. Using cayley-Hamilton theorum, Find A^B if $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

6\. Define Index, signature and Nature

7\. Find the nature of the Quadratic form

$$2x^2+2y^2+2z^2+2yz.$$

8\. Reduce the Quadratic form to matrix form.

$$x^2+4xy+6xz-y^2+2yz+4z^2$$

9\. Reduce matrix Form to quadratic Form $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

10 State cayley's Hamilton theorum.

UNIT-II

EASSY ANSWERS:

1 Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

2 Find the Eigen values and corresponding Eigen vectors of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

3. Determine model matrix of $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{bmatrix}$

also find a) A^8

b) A^4.

4 Show that $A = \begin{bmatrix} i & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix}$ is a skew-Hermitian matrix and the unitory. Find the Eigen values and corresponding Eigen vectors of A.

5. Find the diagnol matrix orthogonally similar to the following real symmetric matrix. Also obtain the transforming matrix $A = \begin{bmatrix} 7 & 4 & -4 \\ 4 & -8 & -1 \\ -4 & -1 & -8 \end{bmatrix}$

6 verify cayley-Hamilton theorum for the matrix

$$A = \begin{bmatrix} 8 & -8 & 2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$

7 verify cayley - Hamilton theorum $\begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$

also find A^4 & A^{-1}

8. Find the nature of quadratic form, index and signature of $10x^2+2y^2+5z^2-4xy-10xz+6yz$.

9 Reduce the quadratic form to the canonical form $2x^2+5y^2+3z^2+4xy$.

10) Reduce the quadratic form $3x^2+5y^2+3z^2-2yz+2zx-2xy$. to canonical from by orthogonal reduction.

11 Reduce the quadratic form $3x_1^2+3x_2^2+3x_3^2+2x_1x_2-2x_1x_3-2x_2x_3$ into sum of squaresform by an orthogonal transformation and give the matrix of transformation.

Short Answer Questions

1. Test for Convergence : $\sum(\sqrt{n^2+1}-\sqrt{n^2-1})$
2. Test for Convergence : $\sum(\sqrt[3]{n^3+1}-n)$
3. Find the nature of the series

$$\frac{3}{4}+\frac{3.6}{4.7}+\frac{3.6.9}{4.7.9}+\cdots\infty$$

4. Test for Convergence of $\sum(\sqrt{n^3+1}-\sqrt{n^3})$
5. Examine the Convergence of $\sum\frac{1}{n(2n+1)}$
6. Examine the Convergence of $\sum\left(\frac{1}{n^{3/2}+n+1}\right)$
7. Test the Convergence of

$$\frac{\sqrt{2}-1}{3^2-1}+\frac{\sqrt{3}-1}{4^2-1}+\frac{\sqrt{4}-1}{5^2-1}+\frac{\sqrt{5}-1}{6^2-1}+\cdots$$

8. Examine the convergence of $\frac{1}{1.3.5}-\frac{1}{3.5.7}+\frac{1}{5.7.9}+\cdots$
9. Examine the convergence of $1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\cdots$
10. Examine the Convergence of $\frac{1}{5.9.13}-\frac{1}{9.13.17}+\frac{1}{13.17.21}+\cdots$

UNIT-III

Long Answer Questions

1. Test the convergence of $\sum \frac{n^4}{n!}$

2. Test the Convergence of the series
$$\sum_{n=1}^{\infty} \frac{1.3.5.\dots.(2n+1)}{2.5.8\dots(3n+2)}$$

3. Test the Convergence of the series
$$\frac{2}{1}+\frac{2.5.8}{1.5.9}+\frac{2.5.8.11}{15.9.13}+\dots+\infty$$

4. Examine the Convergence or divergence of
$$\sum \frac{x^{2n}}{(n+2)\sqrt{(n+2)}},\ (x>0)$$

5. Discuss the Convergence of $\frac{x^{2n}}{(n+2)(\sqrt{n+1})}$, $(x>0)$

6. Discuss the Convergence of $\sum_{n=1}^{\infty} \frac{x^{2n}}{(n+1)(\sqrt{n})}$

7. Find the interval of convergence of the series
$$\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+\dots\infty$$

8. Test for Convergence of $2+\frac{3x}{2}+\frac{4x^2}{3}+\frac{5x^3}{4}+\dots$ $(x>0)$

9. Test for Convergence of the series
$$\frac{x}{1}+\frac{x^3}{2.3}+\frac{1.3x^5}{2.4.5}+\frac{1.3.5.x^7}{2.4.6.7}+\dots$$

10. Test for Convergence of the Series

$$\sum \frac{4.7 \cdots (3n+1)}{1.2.3 \cdots n} x^n$$

11. Examine the convergence of $\frac{1}{3}x^2 + \frac{1.2}{3.5}x^3 + \frac{1.2.3}{3.5.7}x^4 + \cdots (x>0)$

12. Examine the convergence of $\sum \left[\frac{1.4.7 \cdots (3n-2)}{3.6.9 \cdots 3n}\right]^2$

13. Examine the convergence of $\sum \frac{1.3.5 \cdots (2n-1)}{2.4.6 \cdots 2n(2n+2)}$

14. Examine the Convergence or divergence of

$$\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + \cdots \quad (x>0)$$

15. Test the Convergence of the series

$$\sum \left(\frac{2n+1}{n^3+1}\right) x^n, \; x>0$$

16. Test the Convergence of $\sum \frac{(n!)^2}{2n!} x^{2n}$

17. Test the Convergence of $\frac{x^n}{n^{n-1}}$, $(x>0)$

18. Test the Convergence of $\sum \frac{1}{(\log\log n)^n}$

19. Examine the following Series for absolute of conditional convergence

$$\frac{1}{5\sqrt{2}} - \frac{1}{5\sqrt{3}} + \frac{1}{5\sqrt{4}} + \cdots (-1)^n \cdot \frac{1}{5\sqrt{n}} + \cdots$$

20) Test for Convergence of the series

$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

21. Find wheather the following series converges absolutely / conditionally

$$\frac{1}{6}-\frac{1}{6}\cdot\frac{3}{8}+\frac{1.3.5}{6.8.10}-\frac{1.3.5.7}{6.8.10.12}$$

22. Test wheather the following series is absolutely / conditionally convergent $\sum(-1)^{n+1}(\sqrt{n+1}-\sqrt{n})$

23. Examine whether the following series is absolutely / conditionally convergent $1-\frac{1}{3!}+\frac{1}{5!}-\frac{1}{7!}+\cdots$

24. Prove that the series $\sum_{n=2}^{\infty}\frac{(-1)^n}{n(\log n)^3}$ converges Absolutely.

25. Test the following series for absolute / conditional convergence $\sum_{n=1}^{\infty}(-1)^n\frac{\log n}{n^2}$

26. Test the absolute convergence of $\sum(-1)^n\frac{\sin 1/\sqrt{n}}{n-1}$

27. Test wheather the following series is absolutely convergent $\sum_{n=1}^{\infty}\frac{\cos n\pi}{n^2+1}$

UNIT-IV Caluculus.

Eassay Type Questions :

1) Verify Rolle's theorem for the function
$$f(x) = e^{-x/2} x(x+3) \text{ in } (-3, 0)$$

2) Using Mean Value theorem prove that $\boxed{\tan x > x}$ in $\boxed{0 < x < \pi/2}$.

3) If $f(x) = \sqrt{x}$ and $g(x) = 1/\sqrt{x}$ prove that "c' of the cauchy's generalized mean Value theorem is geometric mean of a and b for any $a>0, b>0$

4) Find the region in Which $f(x) = 1-4x-x^2$ is increasing and the region in Which it is decreasing Using Mean Value theorem.

5) Prove that $\pi/6 + 1/5\sqrt{3} < \sin^{-1}(3/5) < \pi/6 + 1/8$

6) Verify Rolle's theorem for $f(x) = \log\left[\frac{x^2+ab}{x(a+b)}\right]$ in (a,b)

7) Verify Taylor theorem for $f(x) = (1-x)^{5/2}$ With lagrange's form of remaind upto 2 terms in $[0,1]$

8) Show that $\frac{\sin^{-1}(x)}{\sqrt{1-x^2}} = x + \frac{4x^3}{3!} + \text{----}$

9) Find the Volume of the Solid generated by the revolution of the area bounded by $y=x^2$ and $y=x$ about y-axis.

10) Find the Volume of the solid When Ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(0<b<a)$ Rotating about Minor axis

11) Find the Volume of the solid generated by revolving the $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(0<b<a)$ (or) $(a>b)$ about major axis.

12) Show that $\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(x+a)^{m+n}} = \frac{\beta(m,n)}{a^n(1+a)^m}$.

13) prove that $\beta(m,n) = \frac{\sqrt{(m)}\cdot\sqrt{(n)}}{\sqrt{(m+n)}}$

14) Express $\int_0^1 x^m(1-x^n)^p dx$ in terms of $\sqrt{}$ functions and hence evaluate $\int_0^1 x^5(1-x^3)^{10} dx$.

15) Evaluate the following

i) $\int_0^1 x^4(\log 1/x)^3 dx$ ii) $\int_0^\infty e^{-x^2} dx$

iii) $\int_0^\infty \sqrt{x}\, e^{-x^2} dx$ iv) $\int_0^{\pi/2} \sin^2\theta \cdot \cos^4\theta \, d\theta$ v) $\int_0^1 \frac{x^3}{\sqrt{1-x}} dx$

16) Evaluate $4\int_0^\infty \frac{x^2}{x^2+1}\,dx$ Using β-function.

17) prove that $\int_0^1 (1-x^n)^{1/n}\,dx = \frac{1}{n}\,\frac{\left[\Gamma(1/n)\right]^2}{2\left[\Gamma(2/n)\right]}$

18) show that $\int_0^1 \frac{x^2}{\sqrt{1-x^4}}\,dx \times \int_0^1 \frac{1}{\sqrt{1-x^4}}\,dx$

Short Answer type Questions :

1) State Generalized mean value theorem.

2) Find the value of c in Rolle's theorem for $f(x) = \sin x$ in $(0, \pi)$.

3) Evaluate $\int_0^{\infty} x^2 e^{-x^4} dx$

4) State Geometric interpretation of Rolle's theorem

5) Explain why mean value theorem does not hold for $f(x) = x^{2/3}$ in $[-1, 1]$

6) Verify generalised mean value theorem for $f(x) = e^x$, $g(x) = e^{-x}$ in $[3, 7]$ and find the value of 'c'.

7) find the volume of the solid generated by revolving the arc of the parabola $x^2 = 12y$ bounded by its latus rectum about y-axis.

8) Find the volume of the solid that result when the region enclosed by the curve $y = x^3$, $x = 0$, $y = 1$ is revolved (that) about the y-axis.

9) find the surface area of sphere generated by the circle $x^2 + y^2 = 16$ about its diameter.

10) Evaluate $\int_0^1 \frac{x^2 \, dx}{\sqrt{1-x^5}}$ interms of Beta function.

11) Evaluate $\int_0^{\pi/2} \sin^5\theta \cdot \cos^{7/2}\theta \, d\theta$

12) find the Value of (i) $\Gamma\left(\frac{5}{2}\right)$ (ii) $\Gamma\left(\frac{-7}{2}\right)$

13) Evaluate $\int_0^{\infty} e^{-x^2} dx$

14) Evaluate $\int_0^{1} \frac{dx}{\sqrt{-\log x}}$

15) Evaluate $\int_0^{1} x^7 (1-x)^5 dx$ by using $\beta-\Gamma$ function.

16) Define Beta function and P.T $\beta(m,n) = \beta(n,m)$

17) Determine the Value of $\beta(2,3)$

18) P.T $\int_0^{\infty} e^{-y^{1/m}} dy = m \cdot \Gamma(m)$.

19) find the Taylor's series expansion of e^x about $x = -1$

20) Verify Rolle's theorem for $f(x) = \frac{1}{x^2}$ in $[-1, 1]$:

UNIT – V Multi Variable Calculus.

Short answer questions :-

1. If $w = (y-z)(z-x)(x-y)$. Find the value of $\frac{dw}{dx} + \frac{dw}{dy} + \frac{dw}{dz}$

2. Verify Euler's theorem for $z = ax^2 + 2bxy + by^2$

3. If $u = x^2 - 2y$, $v = x+y+z$, $w = x-2y+3z$, find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$

4. Find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$. If $u = \frac{x^3y^3}{x^3+y^3}$

5. If $u = \log\frac{x^2+y^2}{x+y}$. S.T $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$

6. If $x = r\cos\theta$, $y = r\sin\theta$, $z = z$, Find $\frac{\partial(r,\theta,z)}{\partial(x,y,z)}$.

   Given that $\frac{\partial(x,y,z)}{\partial(r,\theta,z)} = r$

7. If $u = \log(x^2 + xy + y^2)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2$

8. If $u = \sin^{-1}\left[\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}\right]$, show that $\frac{\partial u}{\partial x} = -\frac{y}{x}\frac{\partial u}{\partial y}$

9. If $z = xy^2 + x^2y$. $x = at^2$, $y = 2at$, find $\frac{dz}{dt}$

10. If $u = y^2 - 4ax$, $x = at^2$, $y = 2at$, find $\frac{du}{dt}$.

UNIT - V

Essay Questions :-

1. If $x = r\cos\theta$, $y = r\sin\theta$, then prove that

$$\frac{\partial r}{\partial x} = \frac{\partial x}{\partial r} \quad \text{and} \quad \frac{1}{r} \cdot \frac{\partial x}{\partial \theta} = r \cdot \frac{\partial \theta}{\partial x}.$$

2. If $x+y+z = u$, $y+z = uv$, $z = uvw$, then evaluate

(i) $\dfrac{\partial(x,y,z)}{\partial(u,v,w)}$ (ii) $J\left[\dfrac{u,v,w}{x,y,z}\right]$

3. If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, show that

$$\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)} = r^2\sin\theta \;\&\; \text{find} \; \frac{\partial(r,\theta,\phi)}{\partial(x,y,z)}$$

4. (i) If $x = \dfrac{u^2}{v}$, $y = \dfrac{v^2}{u}$. Find $\dfrac{\partial(u,v)}{\partial(x,y)}$

(ii) If $x = uv$, $y = \dfrac{u}{v}$, then find $\dfrac{\partial(x,y)}{\partial(u,v)}$

(iii) If $x = uv$, $y = \dfrac{u}{v}$. Verify that $\dfrac{\partial(x,y)}{\partial(u,v)} \times \dfrac{\partial(u,v)}{\partial(x,y)} = 1$

5. Find the maximum and minimum values of

$$x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x = 0$$

6. Find the rectangular parallelopiped of maximum volume that can be inscribed in a sphere.

7. Find the maximum value of $u = x^2 y^3 z^4$, if $2x + 3y + 4z = a$

8. Find the minimum value of $x^2+y^2+z^2$, given $x+y+z = 3a$

9. Find the maximum and minimum values of the function $f(x, y) = x^3y^2(1-x-y)$

10. Show that the functions $u = xy+yz+zx$; $v = x^2+y^2+z^2$; $w = x+y+z$ are functionally related. Find the relation between them.